



Mathematical Discourse That Promotes Conceptual Understanding

by **Elham Kazemi**, UCLA Graduate School of Education & Information Studies

Teachers' actions play a crucial role in challenging students to build a deep understanding of mathematics. When teachers help students build on their thinking, student problem-solving and conceptual understanding increase.

As mathematics teachers, we want students to understand mathematics, not just to recite facts and execute computational procedures. We also know that allowing students to explore and have fun with mathematics may stimulate deep thinking and promote greater conceptual understanding. Providing tasks that are aligned with the National Council of Teachers of Mathematics' (NCTM) 1989 curriculum standards and that are connected to students' lives is only part of what is needed to challenge students to build a deep understanding of mathematics. The teacher's actions play a crucial role.

In a study of upper elementary school classrooms, we observed that when teachers helped students build on their thinking, student problem solving and conceptual understanding were high. I describe here what this "press for understanding" looks like by examining the mathematical activity and discourse in two classrooms— Ms. Carter's and Ms. Andrew's. Students in both classes explored the concept of equivalence and the addition of fractions. They worked on fair-share problems, such as the following:

I invited 8 people to a party (including me) and I had 12 brownies. How much did each person get if everyone got a fair share? Later my mother got home with 9 more brownies. We can always eat more brownies, so we shared these equally too. This time how much brownie did each person get? How much brownie did each person eat all together?

Classroom Similarities: Social Norms

In both Ms. Carter's and Ms. Andrew's classes, we saw students huddled in groups, materials scattered about them, figuring out how to share a batch of brownies equally among a group of people. The students seemed to be engaged in and enjoying their work. Often each group found a slightly different strategy to solve the problem. After moving from group to group, listening to and joining student conversations, both teachers stopped group activity to ask students to share their work and explain how they solved the problem.

Both teachers implemented social norms promoted in the NCTM *Standards*. But practices such as

sharing strategies and collaborating only afford opportunities for students to engage in conceptual thinking; they do not guarantee conceptual thinking.

Classroom Differences: Sociomathematical Norms

We saw important differences between these two teachers in the quality of their students' intellectual engagement with the mathematics. Most of these differences concerned what kind of talk was valued in the classroom and how mathematical concepts are supported. We identified four practices that helped create a high press for conceptual thinking:

- Teachers required that explanations consist of mathematical

continued on page 6

SPRING 1999

Also in this issue:

Learning
Developmental Skills
on the Playground



CONNECTIONS

Publisher

Urban Education Studies Center
Graduate School of Education
& Information Studies (GSE&IS)
UCLA

Acting Director

Rachelle Feiler

Director

Deborah Stipek

Editor

Laura Weishaupt

CONNECTIONS is published twice yearly by the UCLA Urban Education Studies Center at Corinne A. Seeds University Elementary School, Mailbox: 951619, Los Angeles, CA 90095-1619. Contents © The Regents of the University of California, 1999. Portions of this newsletter may be reprinted with our written permission.

UESC Steering Committee 1998-'99

Megan Franke, chair, professor

Norma Feshbach, professor

Cathie Galas, teacher

Ronald Gallimore, professor

Frankie Gelbwachs, education consultant

Anne Gilliland-Swetland, professor

Margaret Heritage, school principal

Carollee Howes, professor

Jan Powell, teacher

Jim Stigler, professor

Sharon Sutton, teacher

Gene Tucker, adjunct professor

Lucil Tuliva, lab school parent

Jill Waterman, lab school parent

Noreen Webb, professor

Rachelle Feiler, UESC interim dean

Harold Levine, GSE&IS interim dean

Deborah Stipek, professor,

UESC director



UCLA

Urban Education Studies Center

Box 951619

Los Angeles, CA 90095-1619

(310) 825-2623

lauraw@ucla.edu

A Message From the Director

A school is more than a collection of classrooms. It is a place where children and those who care about them work together to create opportunities for children to learn. When we think about creating these opportunities our thoughts most often focus on formal instruction in the classroom. For example, this issue contains an article by Elham Kazemi that points out subtle features of reform mathematics instruction that have a significant impact on children's mathematical understanding. Developing the most effective ways to teach children the fundamental skills and knowledge they need is one of the primary responsibilities of schools.

Equally essential in supporting children's learning, however, is providing a school environment that allows them to become confident, independent learners. This year at UES we have implemented a school equity program aimed at creating an environment where children feel comfortable expressing their unique perspectives, talents and strengths. Part of that program is an equity policy that was discussed with all students and their parents early in the school year. Students know there is zero tolerance at the school for verbal harassment, taunts and insults. Children who do not adhere to this policy work with an adult to resolve conflicts and to understand how their words and actions affect others.

The implementation of the equity program has made us more aware of the ways in which children's experiences outside the classroom—in the bathrooms, at lunch and on the play yard—contribute to and affect learning inside the classroom. This issue's other article describes some of the research that Professor Goodwin has conducted on children's language and behavior in play. Her work demonstrates the ways in which children learn social negotiation through interactions with peers at times when adults are not involved. It also illustrates how schools can ensure opportunities for girls to demonstrate expertise and leadership in athletic activities.

In some ways it is easier to focus on the classroom. It seems well within our role as educators to strive to provide the most effective instruction for maximizing student learning. Broadening our vision to include all of children's school experiences as part of their education is a greater challenge. We hope the research presented here will help you achieve both goals.

— Rachelle Feiler

Playing to Learn

by Marjorie Harness Goodwin, UCLA Department of Anthropology

Children's playground games play an important role in the development of language, leadership, organization and social skills. Parents and educators can enhance all children's development by promoting games that interest and challenge both boys and girls.

Most of us know that one of the ways very young children learn is through play, but we sometimes forget this is the case for older children as well. Indeed, a front-page article in the *New York Times* last year reported that many school districts are thinking about eliminating recess because it is felt to be “a waste of time better spent on academics.” Our research shows, however, that organizing games among their peers helps children develop important skills in language, social interaction and organization. Our playground observations also contradict widely held assumptions about differences in the ways that girls and boys communicate and show that communication skills are enhanced by the kinds of interactions that take place during play. The time that children spend on the playground is an important part of their development and learning.

The Study

Researchers and others have argued that middle class girls and boys communicate differently because of an essential difference in the ways that males and females experience the world. Some researchers have claimed that girls are less concerned with competition than boys, and that the language they use is less direct. However, my own ethnographic work looking at interaction among working class primary school African-American children (Goodwin 1990) and bilingual (Spanish/English) working-class second- through fifth-grade girls playing hop scotch (Goodwin 1998)

challenges some of these stereotypical notions of gender differences.

Over a recent five-month period I looked at how fourth-graders in same- and mixed-sex groups use language to construct social organization within a particular activity: the game of jump rope. The site for the study was an elementary school in southern California, which draws children from various parts of Los Angeles and different ethnic groups and social classes. The study was conducted in collaboration with Jill Kushner, Sarah Meacham and Fazila Bhimji, who assisted in videotaping 50 hours of playground interaction.

At the school, girls and boys are not physically separated during recess. They sometimes play separately and sometimes together. On the days we observed, we saw certain patterns among the groups. Fourth-grade boys, for example, alternated between basketball, soccer, volleyball, mischief making, and jump rope. Fourth-grade girls alternated between jump rope, volleyball and talking with friends. Here I analyze the interactions of a specific group of girls, the members of which were from ethnically and economically diverse family backgrounds.

Just Do It: Regulating Activity Through Language

Although jump rope is largely regarded as a girls' game, the school's coaches promoted its merits and consequently both boys and girls frequently played the game during physical education classes and recess. Because both genders were enthusiastic about playing jump rope, the game provides a context for

analyzing how boys and girls organize the activity and how they use language when interacting with peers.

In particular, children's use of directives, or statements designed to get someone else to do something, gives strong clues to their development as negotiators and mediators of conflict. In the children's jump rope sessions, for example, directives and responses to directives affirm and ratify who has the right to make decisions about ways that the game can be played. They are especially powerful in determining *who* can play, which is vital in a game where there is pressure for smaller groups of players.

Important to note in the examples are the ways in which boys and girls interact as they organize games and the effects that their interactions have upon each other. Especially interesting is the way that social roles change over time; as children in the study became more skilled in the activity, the forms of actions they invoked to construct social identities shifted as well. Two jump rope sessions held a month apart show how children practiced giving directives and in the process learned skills for organizing activities, determining social order among their peers and playing leadership roles.

When girls determine how the game is played. The movements accompanying some jump rope rhymes involve great athletic agility. In the game entitled “Texaco Mexico” players must, while the rope is in motion, jump in the air while doing kicks and splits, turning

continued on page 4

Playing to Learn

continued from page 3

around, touching the ground, “paying their taxes” (by slapping the hand of a turner), and “getting outa town” (jumping out of the rope).

In the initial session the fourth-grade boys we studied had little experience with jump rope and generally participated as onlookers to the girls’ game, commenting on the girls’ performances. They complimented the girls on the extraordinary skill required for the intricate moves that accompany the “Texaco Mexico” rhyme.

At the same time the boys provided positive assessments of the girls’ jump rope skills, they distanced themselves from the activity. They ridiculed the chant-like quality of the rhymes using an exaggerated singsong tone of voice. In addition, they mimicked the dance-like movements of the jumpers preparing to jump into a turning rope with exaggerated up-and-down head movements, and laughed while hopping up and down with their wrists cocked. The girls, however, paid little attention to these actions.

An asymmetry of rules. When eventually boys attempted to enter the girls’ game, both boys and girls used their negotiation skills. The boys stated that 1) they did not want to have to jump into the rope while it was turning and 2) they did not want to have to execute the complicated movements involved in “Texaco Mexico.” In response, three members of the four-person girls’ group told the boys they could not play unless they performed these moves. Demonstrating their control of the activity and orchestrating participation in it, the girls used strong directives to counter boys’ statements about how they wanted to play: “You guys will **have** to jump with it,” “You guys will **have** to jump in,” and “You **have** to.”

Texaco Mexico — A Jump Rope Rhyme

Texaco Mexico — Turners turn rope
Went over the hill — Jumper jumps into the moving rope
Where far away — Jumper jumps
They do some splits, splits, splits — Jumper executes splits
And they turn around, round, round — Jumper turns around
And they touch the ground, ground, ground — Jumper touches the ground
And they do some kicks, kicks, kicks — Jumper does kicks
And they pay their taxes, taxes, taxes — Jumper slaps hand of turner
And they get outa town, town, town, — Jumper moves out of the rope
And they jump back in, in, in.— Jumper jumps back into the rope

Throughout the day the girls told the boys what the ground rules were. Boys displayed their subordinate positions by making bids to enter the game through requests, such as, “Can I try it?”

Girls, however, told the boys: “You’re not part of our gang. So you can’t!” One of the girls provided an explicit account for why the boys should leave, telling them: “We don’t want you any more.” Reflecting the girls’ desires rather than the requirements of the activity, this statement is an example of one of the strongest ways of formulating a directive. It demonstrates the girls’ deep confidence in their role as leaders and their power over the game.

Girls Against the Boys: The Contest

During the weeks following the initial session between boys and girls, the boys practiced during recess and eventually became quite skilled in jump rope, although they were not skilled in jumping to the rhyme “Texaco Mexico.” One month after the boys had initially attempted to join the girls’ jump rope group, as seven boys and seven girls were jumping in groups separate from but near each other, the girls challenged the boys to a contest.

Decision-making by boys’ leaders. By the time the contest had been announced, a shift in power had occurred. This was no doubt due at least in part to boys’ deepened interest in the game and their increased confidence in their skills.

Although in the first encounter between girls and boys, the girls made most of the decisions and boys made requests of them, by a month later the boys had considerably more say in what took place. The two best jumpers, Malcolm and Ron, initiated many of the decision-making moves. Once again, language both reflected and determined the speakers’ position of authority. Both boys asserted this position through issuing directives:

(After the girls have practiced several minutes)

Malcolm: All the girls go bye-bye.

(Girls start to move to another area)

Malcolm: Okay. Now the boys get to practice.

Ron: This is our home field.

Two boys with considerably less skill in jumping played the role of gatekeepers, patrolling the group boundaries. They told those who were not ratified participants where to locate their bodies in space:

Lyle: IF YOU’RE NOT IN THE TOURNAMENT GO OVER THERE SOMEWHERE!

Jack: Karl, Stephen, Mark. Is it such a hard decision (sic).

Stay behind the tree.

Now you can watch.

But stay behind the way of the tree.

As these examples illustrate, organizing the game provided an opportunity for defining boundaries of the group and specifying how

continued on page 5

Playing to Learn

continued from page 4

group members implicitly rank with respect to one another, as well as who was excluded.

Girls' decision-making. Although in the second session the boys made many of the decisions, it was the girls who decided two critical features of the game: (1) the specific pairing of individuals in the contest and (2) what rhyme would be used. At the onset there were multiple jumpers inside the rope. Malcolm argued that two girls should compete against two boys simultaneously in the same rope. This was countered by Carleen, who yelled loudly to override the others' talk and assert her definition of the contest as one between rotating girl/boy pairs.

Carleen: No. Okay. We're four girls. Whoever has-
Whoever's last
OKAY. LISTEN!
We're four gi(hh)rls.
WHOEVER GOES LAST
VERSUS whoever's
best on your **team.**

During the first real event in the contest, Malcolm asked the girls to confirm his idea that the event was speed jumping. However, Kesha and Carleen countered his proposal by claiming that the event was Texaco Mexico. This is important in that the rhyme the girls selected is one they were expert in.

Malcolm: What's this one. It's speed jumping?

Carleen: NOOOO.

Ron: What is this

Kesha: It's Texaco Mexic-

Carleen: **No** who could- who could get- Texaco Mexico the longer.

Malcolm: You can't go over a hundred.

Although the boys made multiple imperatives throughout this jump

rope session, it was the girls who defined the important parameters of play. Although the boys beat them in speed jumping, the girls won every contest between a girl and boy where Texaco Mexico was the rhyme being jumped to and they celebrated their victories with hand slaps, victory signs and loud score keeping.

What did we Learn?

By viewing an activity over a month's time we saw that level of skill and "ownership" of the activity, rather than gender, influences how language is used to control the activity and who uses it to do so. Ways of orchestrating social organization through language can shift as group members become more expert.

The girls in this study demonstrated their ability to use imperatives and hold their own during interactions with boys. With practice, boys became more skilled and were no longer in a position of subordination to girls. Yet while the boys made use of imperatives rather than requests in the latter session, it was the girls who initiated the contest frame and set the important ground rules regarding how many players would participate and what the jumping "event" would be. These actions represent a shift toward a more balanced ownership and leadership of the game.

Implications for Educators and Parents

Anthropologists White and Watson-Gegeo have argued that "interpersonal conflict, disagreements, and moral dilemmas are at the heart of social life." Developmental psychologists Shantz and Hartup said that "the virtual 'dance' of discord and accord, of disaffirmation and affirmation... is critical to the comprehension of development... No other single phenomenon plays as broad and significant a role in human development as conflict is thought to."

Our observations illustrate that in organizing playground games, children learn and practice important

language and conflict resolution skills. They express their opinions, negotiate, organize, and socialize; and they do so in ways that differ from what happens in the classroom. When playground activities are omitted from the school day or focus solely on boys' games, both boys and girls miss opportunities to fully develop their skills. Parents and educators must see that children spend time during the school day in playground activities promoting girls' as well as boys' games. Doing so can help boys recognize girls as worthy competitors, increase girls' opportunities to participate in competition, help girls become powerful actors in encounters involving both girls and boys, and expand opportunities for all children to learn important developmental skills.

For Further Information

Goodwin, M. H. (1990). *He-Said-She-Said: Talk as Social Organization among Black Children.*

Bloomington: Indiana University Press.

Goodwin, M. H. (1998). Games of Stance: Conflict and Footing in Hopscotch. In S. Hoyle & C. T. Adger (Eds.), *Kids' Talk: Strategic Language Use in Later Childhood*, (pp. 23-46). New York: Oxford University Press.

Johnson, D. (1998, April 7). Many Schools Putting an End to Child's Play. *New York Times*, A1, A20-A21.

Shantz, C. U. and W. W. Hartup (1992). Conflict and Development: An Introduction. *Conflict in Child and Adolescent Development*. C.U. Shantz and W.W. Hartup. Cambridge, Cambridge University Press: 1-11.

White, G.M. and K.A. Watson-Gegeo (1990). Disentangling Discourse. *Disentangling: Conflict Discourse in Pacific Societies*. K.A. Watson-Gegeo and G.M. White. Stanford, Stanford University Press: 3-49.

Mathematical Discourse

continued from front cover

arguments, not simply procedural summaries of the steps taken to solve the problem.

- Errors were used as opportunities to reconceptualize a problem and explore contradictions and alternative strategies.
- Mathematical thinking involved understanding relations among multiple strategies.
- Collaborative work involved individual accountability and reaching consensus through mathematical argumentation.

Explaining Strategies

The following examples illustrate differences in the two classrooms. First, in Ms. Carter's class, explanations were always linked to *mathematical* reasons. In the following example, Ms. Carter asks Sarah and Jasmine to describe their actions and to *explain why* they chose particular partitioning strategies.

Sarah: The first four we cut in half.

(Jasmine divides squares in half on an overhead transparency.)

Ms. Carter: Now as you explain, could you explain why you did it in half?

Sarah: Because when you put it in half, it becomes four...four... eight halves.

Ms. Carter: Eight halves. What does that mean if there are eight halves?

Sarah: Then each person gets a half.

Ms. Carter: Okay, that each person gets a half. (Jasmine labels halves 1 through 8 for each of the eight people.)

Sarah: Then there were five boxes (brownies) left. We put them in eighths.

Ms. Carter: Okay, so they divided them into eighths. Could you tell us why you chose eighths?

Sarah: It's easiest. Because then everyone will get...each person

will get a half and (addresses Jasmine) "How many eighths?"

Jasmine: (Quietly) Five-eighths.

Ms. Carter: I didn't know why you did it in eighths. That's the reason. I just wanted to know why you chose eighths.

Jasmine: We did eighths because then if we did eighths, each person would get each eighth, I mean one-eighth out of each brownie.

Ms. Carter: Okay, one-eighth out of each brownie. Can you just, you don't have to number, but just show us what you mean by that? I heard the words, but...

(Jasmine shades in one-eighth of each of the five brownies that were divided into eighths.)

Jasmine: Person one would get this...(points to one eighth)

Ms. Carter: Oh, out of each brownie.

Sarah: Out of each brownie, one person will get one eighth.

Ms. Carter: One-eighth. Okay. So how much did they get if they got their fair share?

Jasmine and Sarah: They got a half and five-eighths.

Ms. Carter: Do you want to write that down at the top, so that I can see what you did?

(Jasmine writes $1/2+1/8+1/8+1/8+1/8+1/8$ at the top of the overhead transparency.)

This exchange highlights the conceptual focus of the lesson on fair share. Ms. Carter asked Sarah to explain the importance of having eight halves and why the partitioning strategy using eighths made sense. After Jasmine gave a verbal justification, Ms. Carter continued to press her to link her verbal response to the appropriate pictorial representation and the symbolic representation.

Ms. Andrew's students engaged in the same social practice of sharing their strategies with the class, but the mathematical content of classroom conversations was different. Students shared solutions by giving procedural summaries of the steps they

took to solve the problem, as demonstrated by the following exchange, after Ms. Andrew drew twelve squares on the chalkboard.

(Raymond divides the brownies in half.)

Ms. Andrew: Okay, now would you like to explain to us what...loud...

Raymond: Each one gets one, and I give them a half.

Ms. Andrew: So each person got how much?

Raymond: One and one half.

Ms. Andrew: One half?

Raymond: No, one and one half.

Ms. Andrew: So you're saying that each one gets one and one-half. Does that make sense?

(After a chorus of "yeahs" comes from students, Ms. Andrew moves on to another problem.)

Unlike Ms. Carter, Ms. Andrew did not ask her students to justify why they chose a particular partitioning strategy. Instead, Ms. Andrew often asked questions that required a show of hands or yes/no responses, such as "How many people agree?" "Does this make sense?" or "Do you think that was a good answer?" Ms. Andrew wanted to engage her students in the activity and to see if they understood, but the questions she asked yielded general responses without revealing specific information about the students' thinking.

Reacting to Mathematical Errors

By emphasizing mathematical reasons for actions, Ms. Carter created opportunities for her students to prove that their solutions were correct. She resisted telling students that an answer or reason was wrong, and she invited others to respond to incorrect solutions. Ms. Carter modeled the kinds of questions that may help students think through their own confusion by using their exist-

continued on page 7

This article is reprinted with permission from Teaching Children Mathematics, copyright 1998 by the National Council of Teachers of Mathematics.

Mathematical Discourse

continued from page 6

ing knowledge. Those questions usually involved graphical representations of the fractions. In small groups, students challenged one another when they disagreed on a solution and helped one another find errors.

The interaction among Ms. Carter, Jasmine, and Sarah continued with the following conversation.

(Jasmine writes $1/2+1/8+1/8+1/8+1/8+1/8$ at the top of the overhead transparency.)

Ms. Carter: Okay, so that's what you did. So how much was that in all?

Jasmine: It equals $1-1/8$ or $6/8$.

Ms. Carter: So she says it can equal 6 and $6/8$? (She misheard Jasmine.)

Jasmine: No, it can equal $6/8$ or it can equal $1-1/8$.

Ms. Carter: Okay, so you have two different answers. Could you write them down so people can see it? And boys and girls, I'd like you to respond to what they've written up here. She says it either could equal $6/8$ or $1-1/8$.

(Ms. Carter then proceeds to ask those students who had reasons for their disagreements to share their ideas with their classmates.)

Ms. Carter could have stepped in and pointed out why $6/8$ and $1-1/8$

are not equal. Instead, her response to this mistake was to encourage her students to explore the error.

The mistake also created an opportunity for the entire class to explore contradictions in the solution and to build an understanding of fractional equivalence and the addition of fractions by using an area model. This type of activity and discourse was typical in Ms. Carter's classroom. In a whole-class discussion, each group shared its proof that $1-1/8$ was correct. Neither the students nor Ms. Carter belittled, penalized or discredited anyone who made a mistake. The atmosphere of mutual respect between the students and Ms. Carter allowed the class to think about and build conceptual understandings eagerly.

Ms. Andrew treated errors differently. Note how she provides the mathematical reasoning when three boys explain their solution for sharing five brownies among six people.

Ms. Andrew: They got $1/2$, you already said that. And then $1/6$ and then another sixth. So, how many sixths did they get?

Anthony: One, two.

Ryan: One, two.

Joe: $1/12$.

Ms. Andrew: What did you say? (to Joe) They got two...

Ryan: Sixths.

Anthony: $2/12$.

Joe: $2/6$.

Ms. Andrew: $2/6$ (confirming the right answer). Why did you say $2/12$? Because there are twelve parts altogether?

Anthony: Yeah.

Ms. Andrew: Okay, be sure not to get confused. Because there are two brownies not one. Perfect. Good, good job.

At first, the boys appeared to be guessing the answer to Ms. Andrew's question. She focused on Joe once he said the right answer. Although she predicted accurately why Anthony said $2/12$, she did not ask him to think about why his answer did not work. Instead, she asked and answered the question herself and did not press Anthony to sort out his confusion. Her statement

continued on back cover



Send mailing list information and all other correspondence to:

UCLA/UES
CONNECTIONS Editor
Mailbox: 951619
Los Angeles, CA
90095-1619

Are you on our mailing list?

Do you have a friend or colleague who would like to receive CONNECTIONS? Would you like to be on our mailing list but have not sent in a request? Fill out this form to (check one):

add a friend's name add your name make a correction remove your name from our mailing list

Name _____ Preferred Mailing Address:

Street _____ Apt. or Suite # _____

City _____ State _____ ZIP _____

School/Institution _____ Phone () _____

Position (circle) 1. Teacher: grade level _____ 2. Administrator 3. Researcher 4. Other _____

Mathematical Discourse

continued from page 7

“Because there are two brownies, not one” was left unexplained. As this example illustrates, limited opportunity was available for the members of the group to engage in conceptual thinking about what $1/6$ and $1/12$ signify and how the graphical representation is linked to the numeric representation.

Both Ms. Carter and Ms. Andrew allowed students to make mistakes. That social norm, however, did not press students to examine their work conceptually. Both teachers wanted their students to learn from their mistakes, but Ms. Andrew often supplied the conceptual thinking for her students. In Ms. Carter’s class the students were forced to do the thinking.

Comparing Strategies

Students in both classrooms worked together, shared their strategies, and were praised for their efforts. Students in both classrooms attended to nonmathematical similarities between shared solutions. In Ms. Andrew’s class, strategies were typically offered one after the other, with discussion limited to nonmathematical aspects of students’ work. For example, a pair of students noted that they cut the paper brownies and pasted the pieces under stick-figure illustrations. Another pair had

drawn lines from the fractional parts of the brownies to the individuals that received them. Although the partitioning strategy in both was the same, students viewed the strategies as different because the representations were different. Ms. Carter, however, pressed her students to go beyond their initial observations and reflect on the *mathematical* similarities and differences between strategies.

Accountability and Consensus

In both classrooms, students often worked together to share interpretations and solutions and construct new understandings. Important differences arose between Ms. Andrew’s and Ms. Carter’s classes in the way in which they emphasized individual accountability and consensus. Ms. Carter required her students to make sure that each person contributed to and understood the mathematics involved in the group’s solution. If the students disagreed about an answer, she encouraged them to prove their answers mathematically and to work until they arrived at a consensus. If she noticed that the students were not listening to others during an activity, she reminded them that they had to prove their solutions and that any student in the group must be prepared to discuss the reasons for

the solution in front of the class. As a result, the distribution of work was more equitable. Students listened to one another’s ideas and evaluated their appropriateness before using them.

Ms. Andrew did not describe and discuss collaboration beyond the general directive to “work with a partner” or “remember to work together.” Neither individual accountability nor consensus emerged as topics of discussion in whole-class activity. Typically, only one person would be in control of group work at any particular time and that person would complete most of the work.

Conclusion

We saw a consistently higher press for conceptual thinking in Ms. Carter’s class. She took her students’ ideas seriously as they engaged in building mathematical concepts. In both whole-class discussions and small-group work all students were accountable for participating in an intellectual climate characterized by mathematical argument and justification.

When teachers create a high press for conceptual thinking, mathematics drives not only the activities but the students’ explanations as well. As a result, student achievement in problem solving and conceptual understanding increases.

CONNECTIONS
UCLA/UESC **EE-15**
Seeds University Elementary School
Mailbox: 951619
Los Angeles, CA 90095-1619

Nonprofit Org.
U.S. Postage
PAID
UCLA

address correction requested